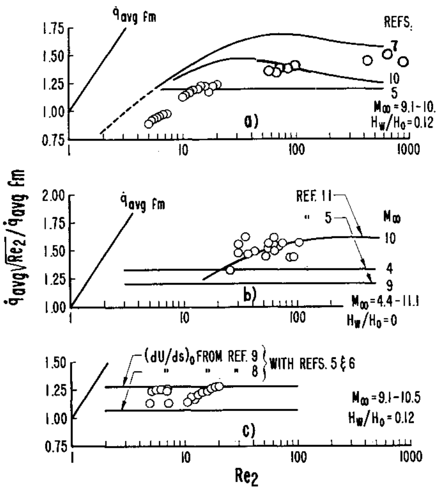

Fig. 1 Average heat-transfer rate parameter: a) hemisphere in nitrogen; b) hemisphere in argon; c) flat-face in nitrogen

References

[1] Potter, J. L., Kinslow, M., Arney, G. D., Jr., and Bailey A. B., "Initial results from a low density hypervelocity wind tunnel," *Hypersonic Flow Research*, edited by F. R. Riddell (Academic Press, New York, 1962), pp. 599–624.

[2] Stollery, J. L., Smith, J. E., and Park, C, "The effects of vibrational relaxation on hypersonic nozzle flows," AGARD Hypersonics Conference, Brussels (1962).

[3] Talbot, L., "Theory of the stagnation-point Langmuir probe," Phys. Fluids **3**, 289–298 (1960).

[4] Potter, J. L. and Miller, J. T., "Experimental heat transfer to blunt axisymmetric bodies near the limit of continuum flow," Arnold Eng. Dev. Center TDR-62-155 (August 1962).

[5] Lees, L., "Laminar heat transfer over blunt-nosed bodies at hypersonic flight speeds," Jet Propulsion **26**, 259–269 (1956).

[6] Vinokur, M., "Laminar heat-transfer distribution on oblate ellipsoidal noses in hypersonic flow," J. Aerospace Sci. **29**, 113–114 (1962).

[7] Cheng, H. K., "Hypersonic shock-layer theory of the stagnation region at low Reynolds number," Cornell Aeronaut. Lab. Rept. AF-1285-A-7 (April 1961).

[8] Chones, A. J., "Heat transfer and pressure measurements on flat-faced flared-tail circular cylinders and normal disks," Naval Ordnance Laboratory NAVORD Rept. 6669 (June 1959).

[9] Probstein, R. F., "Inviscid flow in the stagnation point region of very blunt-nosed bodies at hypersonic flight speeds," Brown Univ. Wright Air Dev. Center TN 56-395 (September 1956).

[10] Probstein, R. F. and Kemp, N. H., "Viscous aerodynamic characteristics in hypersonic rarefield gas flow," J. Aerospace Sci. **27**, 174–192 (1960).

[11] Levinsky, E. S. and Yoshihara, H., "Rarefield hypersonic flow over a sphere," *Hyperssnic Flow Research*, edited by F. R. Riddell (Academic Press, New York, 1962), pp. 81–106.

Approximate Solution of the Energy Equation

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Introduction

In a previous paper by Bush,[1] it has been shown that the incompressible and compressible laminar boundary layer equations may be solved by an approximate method, namely, an iteration procedure. It is the object of this note to extend this method to the solution of the laminar boundary layer energy equation for wedge-type flow with variable wall temperature.

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Table 1 Wall temperature gradient

	$Pr =$ 0.7	$Pr =$ 1.0	$Pr =$ 2.0	$Pr =$ 5.0	$Pr =$ 10	$Pr =$ 20	
$\beta = 0$	0.408	0.467	0.589	...	1.015	1.241	approx.
$\gamma = 0$	0.407	0.470[a]	0.582	...	0.986	1.230	exact
$\beta = 1$	0.468	0.536	...	0.949	...	...	approx.
$\gamma = 0$	0.488	0.560	...	1.011	...	...	exact
$\beta =$ 1.6	0.483	0.556	...	...	...	...	approx.
$\gamma = 0$	0.506	0.583	...	...	...	...	exact
$\beta = 0$	...	1.547	...	...	...	...	approx.
$\gamma = 1$	...	1.625	...	...	...	...	exact
$\beta = 1$	...	0.801	...	...	...	...	approx.
$\gamma = 1$	...	0.812	...	...	...	...	exact

[a] See Ref. 1.

Analysis

The appropriate energy equation for the type of flow and wall temperature mentioned previously is

$$\frac{d^2\theta}{d\eta^2} + Pr\, f\,\frac{d\theta}{d\eta} - Pr(2-\beta)\,\gamma\,\frac{df}{d\eta}\,(\theta - 1) = 0 \tag{1}$$

with the boundary conditions

$$\eta = 0{:}\quad \theta = 0 \qquad \eta \to \infty{:}\quad \theta = 1 \tag{2}$$

where β determines the wedge angle, γ is the power to which the wall temperature is raised, and Pr is the Prandtl number.

Denoting $df/d\eta$ by w, Eq. (1) becomes

$$\frac{d^2\theta}{d\eta^2} = -Pr\,\frac{d\theta}{d\eta}\int_0^\eta w\,d\eta_1 + Pr(2-\beta)\gamma w(\theta - 1) \tag{3}$$

This equation may be solved by iteration by introducing a suitable approximation for θ in the right-hand side. Denoting the resulting solution by G, one has

$$\frac{d^2G}{d\eta^2} = -Pr\,\frac{d\theta}{d\eta}\int_0^\eta w\,d\eta_1 + Pr\,(2-\beta)\gamma w(\theta - 1) \tag{4}$$

The first integral of this equation is

$$\frac{dG}{d\eta} = \int_\eta^\infty Pr\,\frac{d\theta}{d\eta_1}\left(\int_0^{\eta_1} w\,d\eta_2\right)d\eta_1 - \int_\eta^\infty Pr \times (2-\beta)\gamma w(\theta - 1)\,d\eta_1 \tag{5}$$

The second integral of this equation is

$$G(\eta) = \int_0^\eta d\eta_1\left[\int_\eta^\infty Pr\,\frac{d\theta}{d\eta_2}\left(\int_0^{\eta_2} w\,d\eta_3\right)d\eta_2\right] - \int_0^\eta d\eta_1\left[\int_{\eta_1}^\infty Pr\,(2-\beta)\gamma w(\theta - 1)\,d\eta_2\right] \tag{6}$$

satisfying the boundary condition $G(0) = 0$.

If the approximating functions w and θ are

$$w = \mathrm{erf}(a\eta) \qquad a = \text{const} \tag{7a}$$

$$\theta = \mathrm{erf}(b\eta) \qquad b = \text{const} \tag{7b}$$

then the first integration, Eq. (5), yields the temperature gradient at the wall for $\eta = 0$:

$$\frac{dG(0)}{d\eta} = Pr[1 + (2-\beta)\gamma]\left[\frac{(1+m^2)^{1/2} - 1}{a(\pi)^{1/2}}\right]m = \frac{a}{b} \tag{8}$$

and the second integration, Eq. (6), yields

$$G(\infty) = 1 = Pr[1 + (2 - \beta)\gamma]\,\frac{m^2}{\pi a^2}\tan^{-1}m + Pr(2 - \beta)\,\gamma\left\{\frac{m}{2\pi a^2} - \frac{m^2}{4a^2} + \frac{1}{4a(\pi)^{1/2}}\left[m^3 - 1 + \frac{1}{(1 + m^2)^{1/2}}\left(\frac{1 - m^5}{m}\right)\right]\right\} \quad (9)$$

Given β, a^2 may be determined from Ref. 1, Eq. (1.11), and it is

$$a^2 = \tfrac{1}{4} + \beta[\tfrac{1}{4} + (1/2\pi)] \quad (10)$$

For a given Prandtl number Pr and γ, m may be determined from Eq. (9).

Several numerical calculations have been performed, and they are tabulated in Table 1. Comparison between the approximate and Levy's[2] "exact" solutions is made. It may be noted that the results of Ref. 2 are negative but by a transformation may be made to conform to the results of this paper. The nondimensional temperature function of this paper is $\theta(\eta)$. Denoting the temperature function of Ref. 2 by $\theta_1(\eta)$, the relationship between the two functions is

$$\theta = 1 - \theta_1 \quad (11)$$

and, therefore, the derivatives are

$$d\theta/d\eta = -(d\theta_1/d\eta) \quad (12)$$

Although only one iteration was used, the results are quite satisfactory, especially for the case when $\beta = \gamma = 0$.

References

[1] Bush, W. B., "A method of obtaining an approximate solution of the laminar boundary-layer equations," J. Aerospace Sci. **28**, 350 (1961).

[2] Levy, S., "Heat transfer to constant-property laminar boundary layer flows with power-function free-stream velocity and wall-temperature variation," J. Aeronaut. Sci. **19**, 344 (1952).

Continued Comments on the Collapse of Pressure-Loaded Spherical Shells

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During recent years, continued theoretical and experimental interest has been expressed on the collapse of pressure-loaded spherical shells. However, to date there still are important unanswered questions concerning certain discrepancies between tests and the theories or semi-empirical plots. Such comparisons are shown in Figs. 2, 3, and 7 of Refs. 3, 5, and 6, respectively. The spread of data shown in Fig. 2 of Ref. 3 is so large that one wonders whether there really is any rational answer for this phenomenon. Fortunately, the author already has stated in Ref. 1 certain factors contributing to this scatter. Basically, the shell behavior is sensitive to initial irregularities that always exist in practice. The presence of these deviations overshadows any effect of the angle subtended by the shell segment; therefore, this angle is eliminated from the problem. Furthermore, the maximum compressive strain that the shell can sustain is considered as the true measure of the collapse strength.

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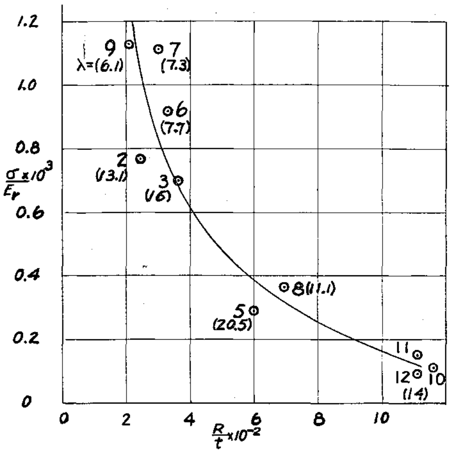

Fig. 1 Plot of data of Homewood, Brine, and Johnson

Based on these considerations, a plot of the test data given in Ref. 3 has been made and is shown in Fig. 1 of the present note. The numbers near the circles are the specimen numbers quoted in Ref. 3. The curve shown is approximately the lowest curve given in Fig. 1 of Ref. 1. The numbers in the parentheses are values of what is believed to be a significant parameter λ used in plotting data in Refs. 3, 5, and 6. It is seen that this parameter does not appear to correlate too well in the present plot. In general, the data presented here appear to be much more consistent and rational than when plotted vs λ.

An effort will be made to interpret the test data given in Refs. 4 and 7 as well as other known data. Then, design curves will be established in light of all these data.

References

[1] Klein, B., "Parameters predicting the initial and final collapse pressures of uniformly loaded spherical shells," J. Aeronaut. Sci. **22**, 69–70 (1955).

[2] Weinitschke, H. J., "The effect of asymmetric deformations on the buckling of shallow spherical shells," J. Aerospace Sci. **29**, 1141–1142 (1962).

[3] Homewood, R. H., Brine, A. C., and Johnson, A. E., Jr., "Buckling instability of monocoque shells," Proc. Soc. Exptl. Stress Anal. *XVIII*, 88–96 (1961).

[4] "Buckling of hemispherical shells under external pressure," Douglas Aircraft Co., Missile Div., SM-38315 (March 1962).

[5] Stambler, I., "Saturn's S-IV stage, designed for 'man-rating'," Space Aeronaut., 52–57 (July 1962).

[6] Budiansky, B., "Buckling of clamped shallow spherical shells," *Proceedings of Iutam Symposium on the Theory of Thin Elastic Shells, Delft* (North-Holland Publishing Co., Amsterdam, 1960), pp. 64–94.

[7] Von Klöppel, K. and Jungbluth, O., "Beitrag zum Durchschlagsproblem dünnwandiger Kugelschalen," Der Stahlbau **22**, 120 (1953).

Transition Relations across Oblique Magnetohydrodynamic Shock Waves

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In magnetohydrodynamics, analogs of the usual gasdynamic shocks occur, and transition relations across hydromagnetic shock waves have been considered by de Hoffman and

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